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On Mr. Gompertz's Method for the Adjustment of Tables of Mortality. By PETER GRAY, F.R.A.S.

WHEN an observation has been made for the purpose of determining the mortality which has prevailed during a specified time in the community observed upon, it is usually found that the series of probabilities of living a year, at the various ages thence deduced, is very far from conforming to our ideas of what an exponent of the law of mortality ought to be. Instead of a series exhibiting throughout its extent a gradual and progressive change of value in passing from each term to the next, we have one showing but a faint approach to the desired regularity. This is just what, from the circumstances in which such observations are usually made, we ought to expect. It is not often that the community subjected to observation is sufficiently large, or that the period during which it is so subjected is sufficiently extended, to furnish us with such a series of *average* results as can alone fitly figure forth the law of mortality.

Now, if our object is only to ascertain the mortality which *has* prevailed during the period to which our observation has been restricted, nothing remains to be done—the series obtained as above *is* a correct representation of that mortality. But there is usually another and ulterior object in view. What we generally want is, a series to serve as a basis of deductions for the future, and which consequently shall exhibit, not the incidental mortality of a limited period, but the *normal* mortality (so to speak) which would have

been found to prevail had the observation been conducted on more extensive bases. Keeping in view, then, that we have no other means of arriving at, or rather of approximating to, the series in question, than by subjecting to treatment the series already obtained, it becomes a matter of importance to determine what is the mode of treatment likely to conduct us to the most satisfactory result. Various methods have been devised and employed by computers who have been engaged in the construction of tables for the *adjustment* of their results, (for so the operation I speak of is termed,) and it is one of these that it is the object of the present paper to explain and illustrate.

The method in question is that which was devised by Mr. Gompertz, and given to the world in the *Philosophical Transactions* for 1825. Although so long before the public, it is far from being so well known as it deserves to be. This may have arisen from various causes. The work containing it is not very accessible; the form in which the investigation of it is given is rendered forbidding by the employment of the obsolete fluxional notation, and a degree of brevity which renders it difficult to be followed; while the whole of the paper containing it is so disfigured by typographical errors as to be in many places almost unintelligible. It is hoped, therefore, that the placing on record here of a description and explanation of the method, with such modifications for increasing the facility of its application as have suggested themselves, will be found useful by cultivators of the science of life contingencies.

In nearly all the methods of adjustment heretofore employed, with the exception of Mr. Gompertz's, the object chiefly kept in view has been, simply (doing, of course, as little violence as possible to the original results) to secure *some sort* of regularity of progression in the series of probabilities of living a year; and this without reference to any *law* in accordance with which this progression *ought* to be regulated. Mr. Gompertz, on the other hand, sets out by proposing for our acceptance a principle with respect to the progression of the rate of mortality, which, on its being admitted, leads to the method of adjustment of which we speak.

There resides in the human frame a certain "power to oppose destruction," which certainly varies with variation in the age; and Mr. Gompertz's principle is, that this power loses equal proportions* in equal times. Now if this be admitted—and it seems by no means unlikely to be true—it will follow that the power to

* The word in Mr. Gompertz's Memoir is, 'portions'; but this is most likely a misprint.

oppose destruction or to avoid death will be denoted by a function of the form mq^x, where x is the time, measured from a specified epoch, and m and q are constants.* Moreover, the *intensity of mortality* at any instant of time is inversely proportional to the power to oppose destruction at the same instant, and it will therefore be denoted by a function of the same form, mq^x, since the reciprocals of exponentials are themselves exponentials.† Now, applying this function to l_x, any number living at age x, we have for the form of the decrement of l_x, in the time dx, $l_x mq^x dx$. That is,

$$dl_x = -l_x mq^x dx\,;$$

$$\therefore \frac{dl_x}{l_x} = -mq^x dx.$$

Hence, integrating,

$$\log.\, l_x = \mathrm{C} - \frac{m}{\log.\, q}\, q^x;$$

consequently, the logarithms being Napierian,

$$l_x = \varepsilon^{\mathrm{C} - \frac{m}{\log.\, q} q^x}$$

$$= \varepsilon^{\mathrm{C}} \,.\, \varepsilon^{-\frac{m}{\log.\, q} q^x}$$

Let $\varepsilon^{\mathrm{C}} = a$, $\varepsilon^{-\frac{m}{\log.\, q}} = b$, and for q write c, and we have finally

$$l_x = ab^{c^x}.$$

Now it is desirable we should carry with us a distinct conception of what we have been doing. We have been investigating, not values, but forms, and the result at which we have arrived is this—namely, that if the function expressing the power to oppose destruction be an exponential, the function expressing the number living will be (so to speak) a *double* exponential: that is, one in which the exponent is itself an exponential. Of the values of the constants which enter the expression we as yet know nothing: in fact, these values will generally be different for different tables, and some of them different perhaps for different parts of the same table. They admit of easy determination, however, for any table, by means of the requisite number of values of the function taken from that table. This I proceed to show.

In its present form the expression is somewhat unmanageable.

* A familiar example of a function of this form is that which denotes the present value of a sum a, due at the end of the time x, namely av^x. This function in the time h loses $av^x - av^{x+h} = av^x(1-v^h)$, which is to the initial value, av^x, in the ratio $1-v^h : 1$, a ratio independent of x.

† Thus the *amount* of a sum a, in the time x, is reciprocally proportional to its *present value* for the same time; and both are exponentials.

It will be more tractable if we put it in the logarithmic form, thus:—

$$\log. l_x = \log. a + c^x \log. b.$$

Since x here is quite general, we may obviously replace it by $x+n$, which gives

$$\log. l_{x+n} = \log. a + c^{x+n} \log. b,$$
$$= \log. a + c^x . c^n \log. b.$$

And since, so long as x does not vary, c^x is constant, we may for log. a write A, and for log. $b . c^x$ we may write B, and the expression finally becomes

$$\log. l_{x+n} = \mathrm{A} + \mathrm{B}c^n.$$

From this expression, by giving to n the values 0, 1, 2, &c., successively, we obviously get the values of log. l_x, log. l_{x+1}, log. l_{x+2}, &c. Thus,

$$\begin{array}{ll} \log. l_x \ = \mathrm{A} + \mathrm{B} & \\ & \mathrm{B}\ (c-1) \\ \log. l_{x+1} = \mathrm{A} + \mathrm{B}c & \\ & \mathrm{B}c\ (c-1) \\ \log. l_{x+2} = \mathrm{A} + \mathrm{B}c^2 & \\ & \mathrm{B}c^2(c-1) \\ \log. l_{x+3} = \mathrm{A} + \mathrm{B}c^3 & \\ \vdots \quad \vdots & \vdots \end{array}$$

Differencing as on the right, and remembering that $\triangle \log. l_x = \log. p_x$, it appears that the logarithmic probabilities of living a year at each age form a series in geometrical progression of which the common ratio is c.

The number of constants in the expression just deduced being three, to determine them we require three values from the table to be adjusted: and it is convenient that these values should be equal distances apart. Thus,

$$\begin{array}{l} \log. l_x \ \ = \mathrm{A} + \mathrm{B} \\ \log. l_{x+n} \ = \mathrm{A} + \mathrm{B}c^n \\ \log. l_{x+2n} = \mathrm{A} + \mathrm{B}c^{2n} \end{array}$$

And, differencing, we find

$$\begin{array}{ll} \triangle \log. l_x \ = \mathrm{B}(c^n - 1) & \\ & \triangle^2 \log. l_x = \mathrm{B}(c^n - 1)^2 : \\ \triangle \log. l_{x+n} = \mathrm{B}c^n(c^n - 1) & \end{array}$$

whence

$$c^n = \frac{\triangle \log. l_{x+n}}{\triangle \log. l_x}, \ \mathrm{B} = \frac{(\triangle \log. l_x)^2}{\triangle^2 \log. l_x}, \text{ and } \mathrm{A} = \log. l_x - \mathrm{B}.$$

The advantage of the method of solution here employed is, that we thereby obtain neater and more commodious expressions for the values of the constants than we should have done had any of the more usual methods of elimination been made use of. The applicability of this method is a consequence of the values selected from the table being equidistant. Had they been otherwise, the solution must have been effected in a different manner.

Preparatory to the application of the formula just deduced to the adjustment of a table, it is requisite to remark that, the table to be adjusted being by hypothesis in an abnormal state—the values not being connected by a recognizable law, otherwise there would be no need for adjustment—it will depend very much upon the selection we make of fundamental values, as we may call them, from the original table, what sort of table we obtain as the result of the adjustment. The less the values selected deviate from the *general run* of the table to be adjusted (so to speak), and the less the distance they are apart in that table, the more closely will the adjusted table resemble the original, and the less violence consequently will have been done to the data.

The number of values to be selected for the determination of the constants is three; and when these do not embrace the entire table (as they will very rarely do), a second selection, or even a third (according to the distance apart of the individual values selected), must be made, the first value of each set being the last of the preceding set. Each set of selected values requires a new determination of the constants. The selected values are retained in the adjusted series; and their occurrence in their places, in the process of formation, is a check on the accuracy of the work.

I propose to exemplify the method of adjustment I have described by showing its application to the Carlisle Table of Mortality. That table, as is sufficiently well known to all who have used it, and as will be immediately apparent to anyone who will take the trouble to cast his eye down the column of "decrements," stands much in need of adjustment. Mr. Milne, its constructor, although he admits that he distributed "the numbers given by the observations among the separate years of age, by a tedious tentative process," and in which he "may have committed error," seems nevertheless to be of opinion that the resulting irregularities "may be indications of a law of nature resulting from our structure."* I do not think that, in the present more advanced state of our knowledge with regard to the law of mortality, many would be found now to agree with him. Be that as it may, I consider the table an exceedingly fit one to be made the subject of experiment, and with that view I have selected it.

I have, without much consideration, taken from the original table, for the fundamental values, the 10th, 20th, 30th, &c.; so that n, which denotes the distance of those values apart, is 10; and the successive steps of the operation will be as follows :—

* *Friendly Societies Report*, 1827, pp. 62, 64.

	Log. l_x.	$\Delta^1(-)$.	$\Delta^2(-)$.	Log. Δ^1.	2 log. Δ^1.	Log. Δ^2.
10	·8102325	·0256152	·0075690	$\bar{2}$·4084978	$\bar{4}$·8169956	$\bar{3}$·8790385
20	·7846173	·0331842		·5209313		
30	·7514331	·0459971	·0162824	·6627304	$\bar{3}$·3254608	$\bar{2}$·2117184
40	·7054360	·0622795		·7943452		
50	·6431565	·0816973	·0993697	·9122077	$\bar{3}$·8244154	$\bar{2}$·9972540
60	·5614592	·1810670		$\bar{1}$·2578393		
70	·3803922	·4012993	·4255053	·6034685	$\bar{1}$·2069370	$\bar{1}$·6289050
80	·9790929	·8268046		·9174029		
90	·1522883					

The first column here contains the logarithms of the numbers living, according to Mr. Milne's table, at the ages 10, 20, &c., as in the margin; the second column contains the first differences of these logarithms; and the third, the second differences of the same—as many, at least, of these latter, as come into use in the computation; the fourth column contains the logarithms of the first differences; the fifth, the doubles of the alternate logarithms in the preceding column; and the sixth, the logarithms of the second differences.*

	Log. c^{10}.	c^{10}.	c.	Log. $(c-1)$.
10	0·1124335	1·295488	1·026227	$\bar{2}$·4187486
30	·1316148	1·353988	1·030769	·4881134
50	·3456316	2·216316	1·082837	·9182244
70	·3139344	2·060319	1·074963	·8748470

The first column here is derived from that of log. Δ^1, by subtracting the first, third, &c. logarithms in that column from the second, fourth, &c., in accordance with the formula $c^n = \frac{\Delta \log. l_{x+n}}{\Delta \log. l_x}$; the second contains the numbers corresponding to the logarithms in the first; the third, the numbers corresponding to one tenth of the same logarithms; and the fourth contains the logarithms of the numbers in the third diminished by unity.

	Log. B.	Log. B $(c-1)$.
10	$\bar{2}$·9379571	$\bar{3}$·3567057
30	$\bar{1}$·1137424	$\bar{3}$·6018557
50	$\bar{2}$·8271614	$\bar{3}$·7453858
70	$\bar{1}$·5780320	$\bar{2}$·4528790

* The first and second differences are negative, and negative numbers have no *arithmetical* logarithms. Nevertheless, the logarithms belonging to such numbers, *considered as positive*, admit of being used for purposes of multiplication and division.

The first column here is derived from the columns 2 log. Δ^1 and log. Δ^2, by subtracting the logarithms in the latter column from the corresponding logarithms in the former, since $\text{B}=(\Delta \text{ log. } l_x)^2 \div \Delta^2 \text{ log. } l_x$; and the second column is formed by adding to the logarithms in the adjoining column the corresponding logarithms in the column log. $(c-1)$.

Having thus got all our auxiliary values (we do not need formally to determine A, as will presently appear), we are now in a condition to proceed to the final formation. It is hereto appended, and to it I now refer.

We saw that $\text{B}(c-1)$, $\text{B}c(c-1)$, $\text{B}c^2(c-1)$, &c., are the logarithmic probabilities of living a year at the successive ages. We form this series by logarithms, therefore, by successive addition of log. c (one tenth part of log. c^{10}) to the initial term, log. $\text{B}(c-1)$. This operation occupies the first column. The initial values just determined, corresponding to ages 10, 30, 50, &c., are first inserted, and the intermediate terms formed by constant addition, by aid of a card, of the proper log. c. On examining the points of junction of the several partial series, it will be noticed that at 30 and 70 they unite very fairly, but that at 50 there is a very awkward break. Various methods might have been used for remedying this, such as selecting other fundamental values; or perhaps applying first Mr. Finlaison's* method of adjustment, as was done by Mr. Galloway in his Amicable Tables: but I have not thought it worth while to take the trouble. Our preliminary values furnish us with the means of going rigorously only as far as age 99. I have extended the table to 102 by using the log. c last determined—that is, I use the same log. c from 70 to 102.

Column (2) contains the numbers corresponding to the logarithms in column 1. Being values of $\text{B}c^n(c-1)$, they are the logarithmic probabilities of living a year in the negative form; for, while c is positive, B is negative, in consequence of its denominator, Δ^2 log. l_x, being negative.

Column (3) contains the logarithms of p_x in the usual mixed form. The change is made by adding -1 to the characteristic, and $+1$ to the mantissa.

* Mr. Finlaison describes his method in his *Report on Life Annuities*, 31st March, 1829. The description seems plain enough, and yet I confess that, on applying it to the data in his 13th and 20th Observations (which are those on which the present Government Annuities are founded), I have not been able to bring out his results. Mine differ widely from his. I have succeeded readily in verifying Mr. Galloway's application of the method, so that I do not think there is room to suppose that I have misapprehended it. There are also, in the tables referred to, several discrepancies between the values in the column of adjusted probabilities and in that headed "Law of Mortality"; but it does not seem worth while pointing them out.

Column (4) contains the logarithms of the numbers living at the successive ages. The fundamental value, log. l_{10}, being inserted, each succeeding value is the sum of the two in the line above it in the same and the next preceding column. Verification of all the preceding work is obtained by comparing log. l_{20}, log. l_{30}, log. l_{40}, &c., as here found, with the fundamental values.

Column (5), containing the numbers of the corresponding logarithms in Column (4), is the adjusted table. I have taken out the numbers to one place more than Mr. Milne, to admit of a more exact estimate of the nature of the graduation. This is more apparent by reference to the next column, which contains the annual decrements. The effects of the breaks here, especially of that at age 50, are very apparent. In Column (7), however, which contains the mean duration of life at each age, the effect of the breaks is hardly traceable; and if interest were involved, they would be still less so. This of course arises from the nature of the function, which takes account of the entire after-lifetime at each age.

For the purpose of comparison, I have given in Column (8) the mean duration according to the original Carlisle Table, and it will be apparent how closely the two sets of values correspond up to about age 87. Beyond that age few will maintain that Mr. Milne's table is a trustworthy guide.

The values of l_x in Column (5) are of course the nearest numbers to five, four, three, &c. (as the case may be) places, corresponding to the logarithms in Column (4). But the converse is not true. The logarithms in Column (4) are not, except in the case of the fundamental values, the *exact* logarithms to the numbers in Column (6). Were a table for actual use formed in the manner of the present table, I should certainly recommend that the logarithms should be altered so as to be in closer accordance with the numbers. If they are not so, there will be discrepancies between the results of different formations which will render the task of construction a very irksome one, as the formulæ of verification will not apply with the requisite degree of closeness. A column D, for example, formed from the logarithms as they at present stand, would not possess the requisite accordance with a column M, formed from the decrements. The fear of thereby doing great violence to the data need not deter from making the small change here suggested. To show this, I set down in parallel columns the logarithms of the curtate mean duration as formed, first from the Column log. p_x, and second from the Column l_x. Up to age 70 the two sets of values are

identical, and beyond that age the difference between them is barely appreciable.

10	·684399	·684399
20	·612832	·612832
30	·530085	·530087
40	·433875	·433877
50	·314499	·314498
60	·141933	·141935
70	·931892	·931893
80	·693421	·693439
90	·409644	·409768

I have been more lengthy in the present paper than I intended to be; but I hope the paper, such as it is, may be found not to be devoid of utility.

x.	(1) Log. $Bc^n(c-1)$.	(2) Log. p_x. (Negative.)	(3) Log. p_x. (Mixed.)	(4) Log. l_x.	(5) l_x.	(6) d_x.	(7) e_x.	(8) e_x. (Carlisle.)
10	$\bar{3}$·3567057	0·0022736	$\bar{1}$·9977264	·8102325	64600	337	48·85	48·82
11	·36794905	23332	76668	·8079589	64263	345	48·10	48·04
12	·37919240	23944	76056	56257	63918	351	47·36	47·27
13	·39043575	24572	75428	32313	63567	359	46·62	46·51
14	·40167900	25216	74784	07741	63208	366	45·88	45·75
15	·41292235	25877	74123	·7982525	62842	373	45·15	45·00
16	·42416570	26556	73444	56648	62469	381	44·41	44·27
17	·43540905	27253	72747	30092	62088	388	43·68	43·57
18	·44665240	27967	72033	02839	61700	396	42·95	42·87
19	·45789585	28701	71299	·7874872	61304	404	42·23	42·17
20	·4691392	29454	70546	46171	60900	412	41·51	41·46
21	·48038255	30226	69774	16717	60488	419	40·78	40·75
22	·49162590	31019	68981	·7786491	60069	428	40·07	40·04
23	·50286925	31832	68168	55472	59641	435	39·35	39·31
24	·51411260	32667	67333	23640	59206	444	38·63	38·59
25	·52535595	33524	66476	·7690973	58762	452	37·92	37·86
26	·53659930	34403	65597	57449	58310	460	37·21	37·14
27	·54784265	35306	64694	23046	57850	468	36·50	36·41
28	·55908600	36231	63769	·7587740	57382	477	35·80	35·69
29	·57032935	37182	62818	51509	56905	485	35·09	35·00
30	·6018558	39981	60019	14327	56420	517	34·39	34·34
31	·61501728	41211	58789	·7474346	55903	528	33·71	33·68
32	·62817876	42479	57521	·7433135	55375	539	33·02	33·03
33	·64134024	43786	56214	·7390656	54836	550	32·34	32·36
34	·65450172	45134	54866	·7346870	54286	561	31·66	31·68
35	·66766320	46523	53477	·7301736	53725	573	30·99	31·00
36	·68082468	47954	52046	·7255213	53152	583	30·32	30·32
37	·69398616	49429	50571	·7207259	52569	595	29·65	29·64
38	·70714764	50950	49050	·7157830	51974	607	28·98	28·96
39	·72030912	52518	47482	·7106880	51367	617	28·32	28·28
40	·7334706	54134	45866	·7054362	50750	629	27·66	27·61
41	·74663208	55800	44200	·7000228	50121	639	26·98	26·97
42	·75979356	57517	42483	·6944428	49482	652	26·34	26·34
43	·77295504	59286	40714	·6886911	48830	662	25·68	25·71
44	·78611652	61111	38889	·6827625	48168	673	25·03	25·09
45	·79927800	62991	37009	·6766514	47495	684	24·38	24·46

x.	(1) Log. $Bc^n(c-1)$.	(2) Log. p_x. (Negative.)	(3) Log. p_x. (Mixed.)	(4) Log. l_x.	(5) l_x.	(6) d_x.	(7) e_x.	(8) e_x. (Carlisle.)
46	$\bar{3}$·81243948	0·0064929	$\bar{1}$·9935071	·6703523	46811	694	23·73	23·82
47	·82560096	66927	33073	·6638594	46117	705	23·08	23·17
48	·83876244	68986	31014	·6571667	45412	716	22·43	22·50
49	·85192392	71109	28891	·6502681	44696	726	21·78	21·81
50	·7453858	55640	44360	·6431572	43970	560	21·13	21·11
51	·77994896	60249	39751	·6375932	43410	598	20·40	20·39
52	·81451212	65240	34760	·6315683	42812	638	19·67	19·68
53	·84907528	70644	29356	·6250443	42174	681	18·96	18·97
54	·88363844	76496	23504	·6179799	41493	724	18·27	18·28
55	·91820160	82833	17167	·6103303	40769	770	17·58	17·58
56	·95276476	89694	10306	·6020470	39999	818	16·91	16·89
57	·98732792	97124	02876	·5930776	39181	866	16·25	16·21
58	$\bar{2}$·02189108	·0105170	·9894830	·5833652	38315	917	15·61	15·55
59	·05645424	13882	86118	·5728482	37398	968	14·98	14·92
60	·09101740	23315	76685	·5614600	36430	1020	14·37	14·34
61	·12558056	33531	66469	·5491285	35410	1072	13·77	13·82
62	·16014372	44592	55408	·5357754	34338	1124	13·18	13·31
63	·19470688	56569	43431	·5213162	33214	1176	12·61	12·81
64	·22927004	69539	30461	·5056593	32038	1227	12·05	12·30
65	·26383320	83583	16417	·4887054	30811	1275	11·51	11·79
66	·29839636	98790	01210	·4703471	29536	1322	10·99	11·27
67	·33295952	·0215258	·9784742	·4504681	28214	1364	10·48	10·75
68	·36752268	33090	66910	·4289423	26850	1403	9·99	10·23
69	·40208584	52398	47602	·4056333	25447	1437	9·51	9·70
70	·4528790	83713	16287	·3803935	24010	1518	9·05	9·18
71	·48427244	·0304980	·9695020	·3520222	22492	1526	8·53	8·65
72	·51566588	27843	72157	·3215242	20966	1524	8·22	8·16
73	·54705932	52419	47581	·2887399	19442	1515	7·82	7·72
74	·57845276	78838	21162	·2534980	17927	1498	7·44	7·33
75	·60984620	·0407235	·9592765	·2156142	16429	1470	7·07	7·01
76	·64123964	37764	62236	·1748907	14959	1435	6·72	6·69
77	·67263308	70580	29420	·1311143	13524	1389	6·38	6·40
78	·70402652	·0505856	·9494144	·0840563	12135	1334	6·05	6·12
79	·73541996	43776	56224	·0334707	10801	1271	5·74	5·80
80	·7668134	84539	15461	·9790931	9530	1200	5·44	5·51
81	·79820684	·0628357	·9371643	·9206392	8330	1122	5·15	5·21
82	·82960028	·0675460	·9324540	·8578035	7208	1038	4·87	4·93
83	·86099372	·0726097	·9273903	·7902575	6170	950	4·61	4·65
84	·89238716	·0780525	·9219475	·7176478	5220	859	4·35	4·39
85	·92378060	·0839037	·9160963	·6395953	4361	766	4·11	4·12
86	·95517404	·0901932	·9098068	·5556916	3595	674	3·88	3·90
87	·98656748	·0969542	·9030458	·4654984	2921	585	3·66	3·71
88	$\bar{1}$·01796092	·1042224	·8957776	·3685442	2336	498	3·46	3·59
89	·04935436	·1120352	·8879648	·2643218	1838	418	3·26	3·47
90	·0807478	·1204336	·8795664	·1522866	1420	344	3·07	3·28
91	·11214124	·1294617	·8705383	·0318530	1076	277	2·89	3·26
92	·14353468	·1391665	·8608335	·9023913	799	219	2·72	3·37
93	·17492812	·1495988	·8504012	·7632248	580	169	2·56	3·48
94	·20632156	·1608132	·8391868	·6136260	411	127	2·40	3·53
95	·23771500	·1728682	·8271318	·4528128	284	93	2·26	3·53
96	·26910844	·1858268	·8141732	·2799446	191	67	2·11	3·46
97	·30050188	·1997569	·8002431	·0941178	124	46	1·98	3·28
98	·33189532	·2147313	·7852687	·8943609	78	30	1·84	3·07
99	·36328876	·2308282	·7691718	·6796296	48	20	1·69	2·77
100	·3946822	·2481315	·7518685	·4488014	28	12	1·53	2·28
101	·42607564	·2667323	·7332677	·2006699	16	7	1·32	1·79
102	·45746908	·2867273	·7132727	·9339376	9	5	1·02	1·30
103	. . .	. .	. .	·6472103	4	4	·50	·83